

Intermittency and Localization

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Glossary

- **Auto-catalysis.** In systems that go through several reactions, the reaction is called **autocatalytic** if the reaction product is itself the catalyst for that reaction.
- **Exponential growth.** An autocatalytic reaction is usually described with a simple linear, first order differential equation. The solution for it is an exponential increasing/decreasing function.
- **Logistic growth.** If one adds a saturation term (of power two) to the linear first order differential equation which describes exponential growth, the resulting solution saturates instead of ever-lasting grow. The solution to this system is described with a **logistic curve** and the system is said to follow a **logistic growth**.
- **Reaction-diffusion systems.** Reaction-diffusion systems are mathematical models that describe how the concentration of one or more substances distributed in space changes under the influence of two processes: local reactions in which the substances are converted into each other, and diffusion which causes the substances to spread out in space.

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1 Definition

In this paper, we show how simple logistic growth that was studied intensively during the last 200 years in many domains of science could be extended in a rather simple way. The resulting extended model has, among other features, two very important ones: Intermittency and Localization. These features were observed repeatedly along the history of science in an enormous number of real-life systems in Economics, Sociology, Biology, Ecology and more. We suggest by this a unified theoretical "umbrella" that might serve in a surprising way many scientific disciplines who share similar observed patterns.

2 Introduction

A well known joke, that many physicists like to tell during their talks in order to demonstrate the strength of simplifying the problem one has in hand is: "First, let us consider a spherical cow...". Although, no one really believes in spherical cows - the power of simplification is well accepted and appreciated by the Physics community, or as Albert Einstein put it, very accurately: "Everything should be made as simple as possible, but not simpler".

There are many more such "mantras" like: "Keep it simple, stupid", "Kill your darlings" and "Less is more". As these lines of thought were adopted so strongly by physicist for so much time, the statement of P.W. Anderson that "More is different" made such a revolution in Science. In the paper that has this title, Anderson pushed the new scientific (inter-) discipline, now known as "complexity". By introducing these new ideas, Anderson paved the way for many physicists carrying with them heavy weapons from traditional physics to start thinking and attacking many problems from a variety of scientific disciplines.

A lot of criticism about such physicists that try to cross the borders of their discipline is about *over-simplifying* real-life problems in order to be able to solve the resulting models with the tools they already have. Due to that, it is important to emphasize here that by working inside the framework of complexity one tries not to lose the minimal theoretical ingredients of the problem that are sufficient to produce the complex observed outcome. Rather than this, one tries to study to the best of one's ability, the simplest *possible* model.

A common question that arises in the social sciences is: *Why Improbable Things are so Frequent?*: Fine-tuned irreducibly complex systems have generically a low probability to appear and highly integrated \simeq arranged systems are usually "artificial" (often man-made) and untypical. Yet many complex systems are found lately to be "self-organized". More precisely, the amount of non-generic, fine tuned and highly integrated systems is much larger in nature from what would be reasonably expected from generic stochastic estimations. It often happens that even though the range of parameters necessary for some nontrivial collective phenomenon to emerge is very narrow (or even an isolated single "point" out of an continuum infinite range), the phenomenon does actually take place in nature. This leads to collective objects whose properties are not explainable by the generic dynamics of their components. The explanation of the generic emergence of systems which

are non-generic from the Multi-Agent point of view seems to be related to self-catalyzing dynamics.

As suggested by the examples above, the frequency with which we encounter non-generic situations in self-catalyzing systems is not so surprising. Consider a space of all possible systems obtainable from certain chemical and physical parts. Even if a macroscopic number of those systems are not auto-catalytic and only a very small number happen to be auto-catalytic after enough time, one of the auto-catalytic systems will eventually arise. Once this happens, the auto-catalytic system will start multiplying leading to a final (or far-future) situation in which those auto-catalytic - a priori very improbable systems - are "over-represented" compared with their "natural" probability of occurrence. Basically, this is how life spread all over Earth.

In this paper, we show how simple logistic growth that was studied intensively during the last 200 years in many domains of science could be extended in a rather simple way and with these extensions is capable to produce a collection of behaviors widely observed in an enormous number of real-life systems in Economics, Sociology, Biology, Ecology and more. For other reviews in this direction we recommend on [38, 4, 18]

The paper will start with a historical overview of the use of logistic-like systems in science since its introduction by Malthus in 1798 until today. The next section will present a view of the minimal, though sufficient, extensions to the classical logistic system that are able to bring this theoretical framework, closer to reality, but "auto-catalysis" yet still solvable analytically in many regions of the parameter's space. Then, we will show some of the successes we had in applying this framework to real-life systems. We will finish the paper by a short fantasy trying to describe a dream about the possible usages of this powerful theoretical framework in the so called "soft" sciences in the future.

3 Logistic systems: From Malthus until today.

3.1 "auto-catalysis"

One of the key concepts underlying the emergence of complex macroscopic features is auto-catalysis. We therefore give at this point a provisory definition of it: auto-catalysis = self-perpetuation, \simeq reproduction, \simeq multi-

plication. As opposed to the usual stochastic systems in which the microscopic dynamics changes typically the individual microscopic quantities by additive steps (e.g. a molecule receiving or releasing a quantum of energy), the auto-catalytic microscopic dynamics involve multiplicative changes (e.g. the market worth of a company changes by a factor (index) after each elementary transaction). Such auto-catalytic microscopic rules are widespread in chemistry (under the name of auto-catalysis), biology (reproduction \simeq multiplication, species perpetuation), social sciences (profit, returns, rate of growth).

The "autocatalytic" essence of the growth processes was formally expressed as early as 1798 by T.R. Malthus[24] who wrote a differential equation for describing the dynamics of a population of proliferating individuals:

$$\frac{dW(t)}{dt} = a \cdot W(t) \quad (1)$$

The growth rate of the population W is proportional to W itself and parametrized by a relative growth (/proliferation) rate a . The Malthus equation can be reinterpreted to represent a very wide range of phenomena in various fields: behavior adoption in sociology, proliferation in biology, capital returns in economics, or proselytizing in politics. The (exponential) solution $\sim e^{(a \cdot t)}$ of this equation influenced much of the subsequent ideas in various fields and in particular it roused the first worries about the sustainability of growth. Malthus himself expressed great concern of the humanitarian "catastrophe" that unlimited population growth may lead to. However, Verhulst [39] introduced (in 1838) a nonlinear interaction term $-b \cdot W^2$ (that may represent (confrontation over) limited resources in biology, competition in economics, limited constituency in politics and finite population in sociology)

$$\frac{dW(t)}{dt} = a \cdot W(t) - b \cdot W^2(t) \quad (2)$$

By including this term, rather than increasing indefinitely, the solution saturates at a constant asymptotic value $W \longrightarrow \frac{a}{b}$. For the following two centuries, this "logistic dynamics" was considered by the leading scientists as a crucial element in various fields from biology (Volterra[40]) to "the everyday world of politics and economics" (Lord May[26]).

3.2 Real-Life examples

3.2.1 The A(utocatalysis)-Bomb

The first and the most dramatic example of the macroscopic explosive power of the Multi-agent auto-catalytic systems is the nuclear (Atom) bomb. The simple microscopic interaction underlying it is that the U235 nucleus, when hit by a neutron splits into a few energetic fragments including neutrons:



On the basis of (autocatalysis equation 1) even without knowing what is a neutron or a U235 nucleus, it is clear that a macroscopic reaction chain may develop: if there are other U235 nuclei in the neighborhood, the neutrons resulting from the first (autocatalysis equation 1) may hit some of them and produce similar new reactions. Those reactions will produce more neutrons that will hit more U235 that will produce more neutrons..

The result will be a chain (or rather "branching tree") of reactions in which the neutrons resulting from one generation of fission events induce a new generation of fission events by hitting new U235 nuclei. This "chain reaction" will go on until eventually, the entire available U235 population (of typically some 10^{26} nuclei) is exhausted and their corresponding energy is emitted: the atomic explosion. The crucial feature in the equation above, which we call "auto-catalysis", is that by inputting one neutron n in the reaction one obtains two (or more) neutrons ($n+n$). The theoretical possibility of iterating it and have an exponentially increasing macroscopic number of reactions was explained in a letter from Einstein to President Roosevelt. But only the later attack on Pearl Harbor lead to the initiation of the Manhattan project and the eventual construction of the A-bomb.

It is not by chance that the basic Multi-Agent method (the Monte Carlo simulation algorithm used until this very day in physics applications) was invented by people (Metropolis, Rosenbluth, Rosenbluth, Teller, Teller) involved in nuclear weapons research: the Multi-Agent method is the best fit method to compute realistically the macroscopic effects originating in microscopic interactions!

3.2.2 The B-Bomb: autocatalysis and localization in immunology

In no field is the auto-catalysis and localization more critical than in the emergence of living organisms functions out of the elementary interactions of

cells and enzymes. From the very beginning of an embryo development the problem is how to create a "controlled chain reaction" such that each cell (starting with the initial egg) divides into similar cells, yet spatio-temporal structures (systems and organs) emerge. Let us consider the immune system as an example. The study of the Immune System for the past half century has succeeded in characterizing the key: cells, molecules, and genes. As always in complex systems, the mere knowledge of the microscopic world is not sufficient (and, on the other hand, some details of the micro are not necessary). Understanding comes from the identification of the relevant microscopic interactions and the construction of a Multi-Agent Simulation with which to demonstrate in detail how the complex behavior of the immune system emerges. Indeed, the immune system provides an outstanding example of the emergence of unexpectedly complex behavior from a relatively limited number of simple components interacting according to known simple rules. By simulating their interactions in computer experiments that parallel real immunology experiments, one can check and validate the various mechanisms for the emergence of collective functions in the immune system. (E.g. recognition and destruction of various threatening antigens, the oscillations characteristic to rheumatoid arthritis, the localization of diabetes 1 to pancreatic islets etc). This would allow one to design further experiments, to predict their outcome and to control the mechanisms responsible for various auto-immune diseases and their treatment.

3.2.3 The Tulip Bomb.

The tulip mania is one of the most celebrated and dramatic economic bubbles in history. It involved the rise of the tulip bulb prices in 1637 to the level of average house prices. In the same year, after an increase by a factor of 20 within a month, the market collapsed back within the next 3 months. After loosing a fortune in a similar event (triggered by the South Sea Co.) in 1720 at the London Stock, Sir Isaac Newton was quoted to say, "I can calculate the motions of the heavenly bodies, but not the madness of people."

It might seem over-ambitious to try where Newton has failed but let us not forget that we are 300 years later, have big computers and had plenty of additional opportunities to contemplate the madness of people. One finds that global "macroscopic" (and often "catastrophic") economic phenomena are generated by reasonably simple buy and sell "microscopic" operations. Much attention was paid lately to the sales dynamics of marketable products.

Large amounts of data has been collected describing the propagation and extent of sales of new products, yet only lately one started to study the implications of the autocatalytic multi-agent reaction-diffusion formalism in describing the underlying microscopic process. [41, 37, 12, 42]

3.3 Extensions of the classical logistic system.

One of the great early successes of the logistic dynamics was its application to the spread of malaria in humans and mosquito's. Sir Ronald Ross was awarded the Nobel prize[30] for this work. His ideas were expressed by Lotka[19] in terms of a coupled system of two equations generalizing (2):

$$\begin{aligned} dw_1(t)/dt &= a_1 \cdot w_1(t) + a_{12} \cdot w_2(t) - a_{112} \cdot w_1(t) \cdot w_2(t) \\ dw_2(t)/dt &= a_2 \cdot w_2(t) + a_{21} \cdot w_1(t) - a_{212} \cdot w_1(t) \cdot w_2(t) \end{aligned} \quad (4)$$

Lotka has studied numerically this system in order to predict the ratios between the infected mosquitoes and the infected humans and the stability of the system. Vito Volterra advocated independently the use of equations in biology and social sciences [40] and re-deduced the logistic curve by reducing the Verhulst equation (2) to a variational principle that maximized a function that he named "quantity of life"[19]. Later, R.A. Fisher[11] extended of (2) to spatial distributed systems and expressed it in terms of partial differential equations:

$$\frac{\partial W(\vec{x}, t)}{\partial t} = a \cdot W(\vec{x}, t) - b \cdot W^2(\vec{x}, t) + D \cdot \nabla^2 W(\vec{x}, t) \quad (5)$$

He applied this to the spread of a mutant superior gene within a population and showed that as opposed to usual diffusion, the propagation consists of a sharp frontier ("Fisher wave") that advances with constant speed (rather than proportional to \sqrt{t} as in usual diffusion). Following its formulation, the mathematical study of (5) was taken over by mathematicians[17] and lead eventually a large number of physics studies (especially on the anomalous and fractal properties of the interface [1, 14, 15, 6]).

A crucial step was then taken by Eigen[9] and Eigen and Schuster[10] who generalized the Lotka system (3) of 2 equations for 2 populations to an arbitrary number of equations \simeq populations. They used the new system in the study of the Darwinian selection and evolution in prebiotic environments. More precisely, they considered "quasi-species" of auto-catalytic (self reproducing RNA sequences) molecules which can undergo mutations. Each

sequence i self-replicates at a rate a_i and undergoes mutations to other sequences j at rates a_{ij} . The resulting system of equations is:

$$dW_i(t)/dt = a_i W_i(t) + \sum_{j=1}^N a_{ij} W_j(t) - \sum_{j=1}^N a_{ji} W_i(t) - b(\overrightarrow{W(t)}, t) W_i(t) \quad (6)$$

The arbitrary function $b(\overrightarrow{W(t)}, t)$ represents generically the interaction with the environment (in the specific case of ref [10] the result of replenishing and stirring the container continuously).

The extension of the logistic framework to social sciences was strongly advanced by Elliot Montroll who based a book on social dynamics on the principle that "almost all the social phenomena, except in their relatively brief abnormal times obey the logistic growth." [27].

An analogy that was often exploited in economics was the ecology-market metaphor (e.g. [31]) which was advanced in parallel with the more mechanical physics analogies. The connection to the logistic framework was strengthened by the evolutionary economics metaphor (e.g. [28, 8, 16]). This led to the extension of (6) to economics with the a_i 's representing capital \simeq GDP growth rates and the a_{ij} 's representing trade, social security, mutual help or other mechanisms of wealth transfer (e.g. taxes \simeq subsidies). More recently [25] the logistic dynamics was applied to the dynamics of the equities i within a personal portfolio. Then $a_i(t)$'s are interpreted as the rate of growth of the equity i (at time t) and a_{ij} as the periodic redistribution of capital between the equities by the owner of the portfolio (in order to optimize it). Stochastic generalizations of the logistic \simeq Lotka-Volterra equations were studied also in a large body of mathematical literature (e.g. [17]), and in order to get meaningful results out of the model, one has to introduce the noise in a proper way that will stand for its effect in real-life systems.

3.4 The danger of being mean - Simple examples.

In this sub-section we argue why microscopic (i.e. agent-based) studies are needed and why simplification in the style of mean field theories can be seriously wrong.

If we deal with a small biological population, then due to random accidents it may die out completely and irreversibly. For example, poachers may kill the two surviving males of a small elephant herd which is isolated from other elephants. It does not help the herd if one shows that *on average* there is enough food and space for two adult males, two adult females, and

several calves. For larger populations usually such extreme fluctuations are less probable, and the time until it happens may increase exponentially with the population size.

Also, a hurricane may sink a ship even if averaged over the whole Atlantic Ocean the absolute value of the wind speed and wave height are moderate. In a marriage a husband is supposed to be faithful to his wife and should not average his efforts to become a father over 10^9 women; at least that's what wives often demand.

A less trivial example is demography. If you want to know how many people of retirement age are there for every thousand people of working age, usually one takes into account mortalities, birth rates, and migration. Let us assume, however, that one group of the population has a higher birth rate than the rest and that this difference is given on to the following generations, either genetically or culturally. Then, if everything else is the same, the group with the higher birth rate will finally dominate in the population, and using the *average* birth rate is not correct. (Of course, if the difference is small and we want to extrapolate over less than a century, then the average birth rate is still a good approximation.) One could remedy this error by simulating the two populations together; but then there could be other inherited traits which are demographically relevant, and thus with more and finer subdivisions we finally end up with agent-based demography [5], dealing with each individual.

This explains the conceptual gap between sciences: in conditions in which only a few exceptional individuals dominate, it is impossible to explain the behavior of the collective by plausible arguments about the typical or most probable individual. In fact, in the emergence of nuclei from nucleons, molecules from atoms, DNA from simple molecules, humans from apes, there are always the un-typical cases (with accidentally exceptional advantageous properties) that carry the day. This effect seems to embrace the emergence of complex collective objects in a very wide range of disciplines from bacteria to economic enterprises, from emergence of life and Darwinism to globalization and sustainability.

In the following section we will bring examples [33] where these effects lead to strong localization, such that the mean-field approximations give qualitatively wrong results, like predicting extinction where survival is possible. The approximations do not become good if only the population is large.

In conclusion, generic logistic ideas hinted by (2) arose for the last century in an extremely wide-ranging set of applications. For each discipline, subject and system, the variables of the model had to be interpreted in terms of the

empirical observables and adapted to the relevant range of parameters and initial conditions. Once the parameters are specified, the generic framework (6) (plus noise) becomes a well defined model for a specific system. Then, one can derive from it precise predictions and confront them with the data.

4 Minimal extensions to the classical logistic system.

Here, we show how by restricting the parameter's regime of the generic framework (6) (plus noise), one ends up with a model that has a very strong prediction's power.[3, 2, 18, 29, 35, 36, 34]

4.1 Case 1: The Generalized Lotka-Volterra System

If one considers a uniform interaction in (6), the resulting equation can be written as:

$$dW_i(t)/dt = a_i \cdot W_i(t) + \alpha \cdot W(t) - b(\overrightarrow{W(t)}, t) \cdot W_i(t) \quad (7)$$

where $W(t)$ is the average value of the W_i 's ; then it was shown [18, 29, 35, 36, 34] that :

- The system has a steady state for the normalized quantity $X_i(t) \equiv \frac{W_i(t)}{W(t)}$
- The steady state distribution of the X_i could be calculated analytically and the resulting distribution has the following form: $P(X) = e^{-2\alpha/XD} \cdot X^{-2-2\alpha/D}$ where D is the variance of the distribution from which the growth rates (a_i 's) is drawn out of.
- The fluctuations of the average ($W(t)$) have a wide distribution with a power-law tail that is closely connected with the value of the steady state distribution $(-2 - 2\alpha/D)$

Obviously, as there is no explicit space in this system, one cannot see localization effects. However, intermittency is very clear here: The fluctuations of the average value are enormous but changing around a fixed value. The possible interpretation of such a model are very diverse:

- Income Distribution: $W_i(t)$ can represent the annual income of each individual in the society - then, the $W\alpha$ term is connected to social benefits one gets from the being part of the society, such as social security, charity and minimum wage. The a_i 's stand for the relative change between this year and the previous one. $b(\overrightarrow{W(t)}, t)) \cdot W_i(t)$ then represents the overall trend of the market - periods of depression and of external investments.
- Stock Market: $W_i(t)$ can represent the value of a specific stock in the stock market (at the closing time of the market for example) - then, the $W\alpha$ term is connected to correlations among the different stocks in the market. The a_i 's stand again for the relative change between the value today and the previous one. $b(\overrightarrow{W(t)}, t)) \cdot W_i(t)$ represents the overall trend of the market - periods of depression and of external investments.
- Population Dynamics: $W_i(t)$ can represent the number of individuals from a specific species in animals or of a specific nation in humans - then, the $W\alpha$ term is connected to immigration or mutations connecting the different populations. $b(\overrightarrow{W(t)}, t)) \cdot W_i(t)$ represents the conditions for breeding.

There are many more possible interpretations but the point is clear. For each interpretation one can argue that the uniform choice of the interaction matrix is unrealistic - of course it might be true, but as it turns out lately, the power-law prediction is very robust and can stand many different choices of this matrix.

4.2 Case 2: The "AB Model"

The "AB Model" [33, 32, 23, 22, 20] is actually a reaction-diffusion system which has two types of agents: A and B . It is a discrete system, both in space and in the fields it describes (A and B in any spatial point are natural numbers, never negative) and as such needs to be described with a set of rate equations. Then the agents may go through the following possible processes with the corresponding rates:

- Diffusion: at each time step, with probabilities $D_a/2d$ and $D_b/2d$, respectively, an A or B moves to a nearest neighbour site on a d -dimensional lattice.

- Reaction: at each time step, with probabilities μ and $\lambda \cdot N_A$, a single B dies or gives birth to a new B , respectively, where N_A is the number of A 's in the same location.

Naively, this system can be mapped into two partial differential equations:

$$\frac{dB(x,t)}{dt} = D_b \cdot \nabla^2 B(x,t) + (\lambda \cdot A(x,t) - \mu) \cdot B(x,t) \quad (8)$$

$$\frac{dA(x,t)}{dt} = D_a \cdot \nabla^2 A(x,t) \quad (9)$$

It is tempting to say that we can solve equation (9) to get :

$$A(x,t) \longrightarrow n_A \quad (10)$$

in long times and then to plug it into equation (8) to say that depending on the parameter $m \equiv (n_A \cdot \lambda - \mu)$ the total number of B 's will either increase exponentially (if $m > 0$) or decrease exponentially (if $m < 0$). It turns out that this "mean-field" treatment is totally wrong and as was shown in [33] in low enough dimensions ($d \leq 2$) the B 's will asymptotically increase exponentially no matter what the rest of the parameters are! The intuitive explanation for this surprising result is that the B 's somehow adapt themselves to be localized around regions with good conditions (large number of A_i). One can see a typical snapshot of this system in Figure 1. Another prediction[23] of this model is the intermittency of the total number of B 's even when one adds a saturation term similar to the second term in equation (2). Yet one more prediction of this model is the "J-shape" in the total number of B 's: i.e. initial decline followed by lasting exponential growth, Figure 2.

5 Applying these models to real-life systems.

As mentioned in 3.2, many real-life systems have characteristics that can be explained with the "AB Model" or the "GLV":

In the immune system, it was shown in [21] that the body's B cells tend to grow in "places" in the genome space where they are needed (depending on the diseases existing in the system).

In the Internet, it was shown in [13] that one can use the theoretical understanding of the model in order to plan strategies that will improve the way

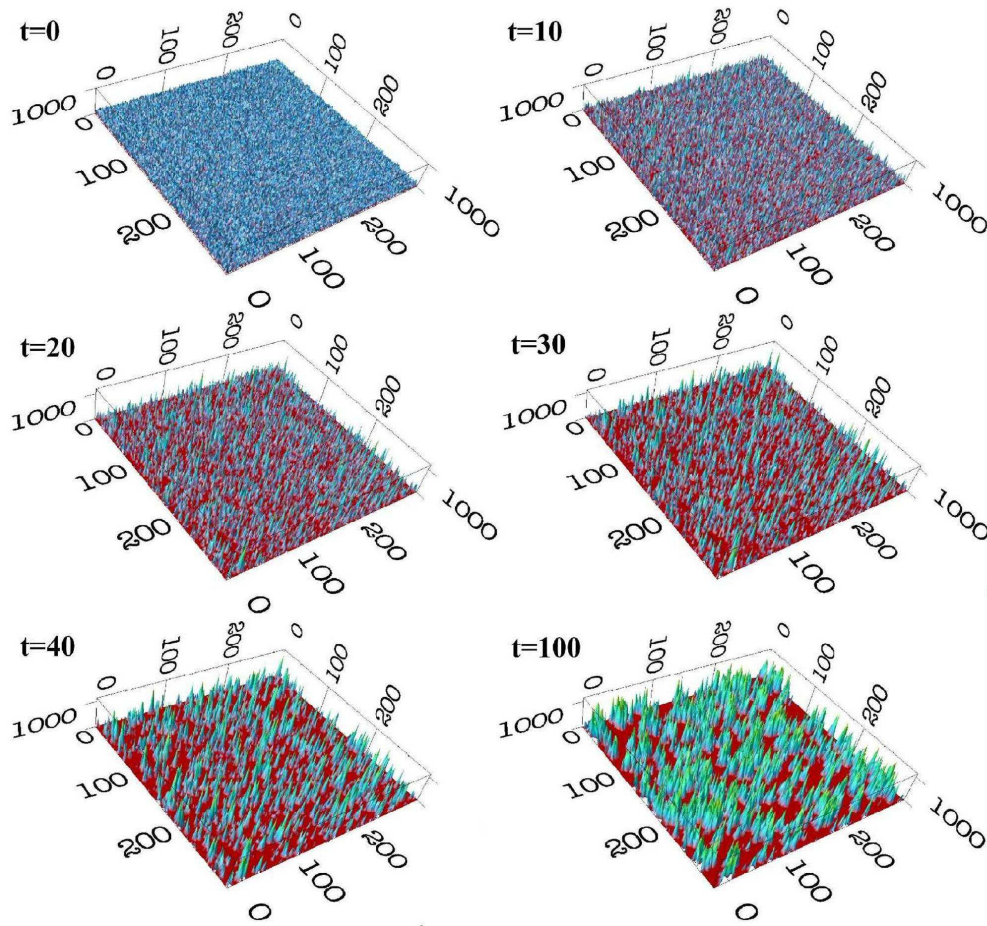


Figure 1: **Snapshots of the "AB Model" in two dimensions [Log scale]** in these 6 snapshots the time evolution of the "AB Model" is demonstrated: The two dimension lattice is set in $t = 0$ to be in a random distribution of the B 's drawn from a Poisson distribution. The parameters are set in a way that if one looks at the mean-field approximation one will guess that the system needs to go to extinction in a relatively short time. However, due to the discreteness of the catalysts and the reactants, the B 's adapt themselves to the rich (in food) areas and the famous "Islands" structure is formed.

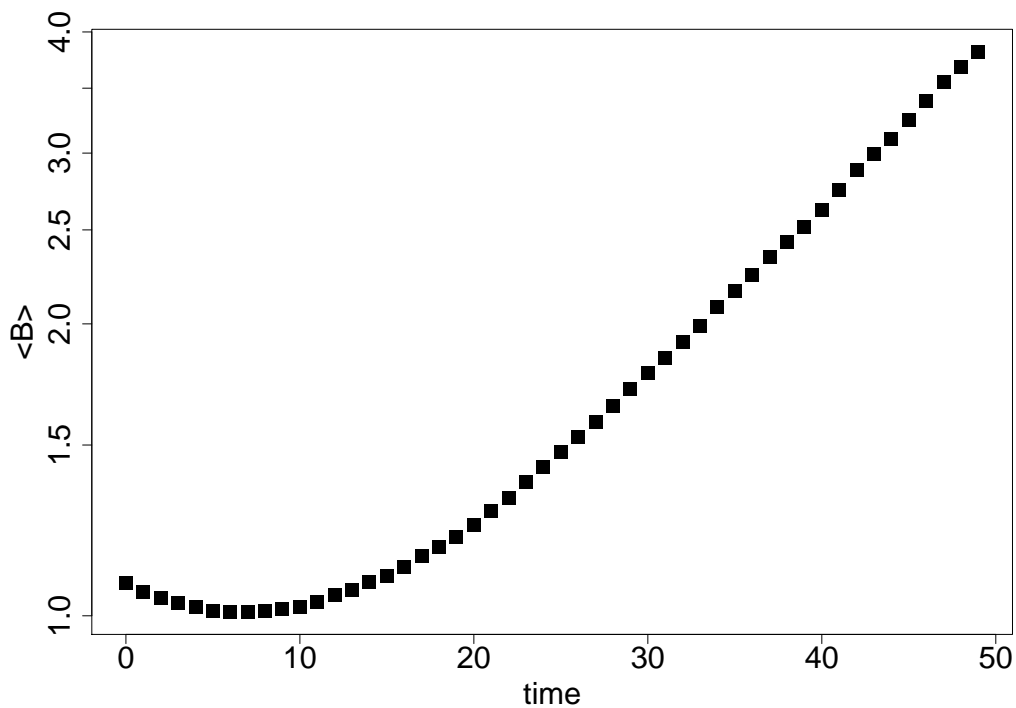


Figure 2: **The time evolution of the average B number in the "AB Model" in two dimensions** For the same parameters as Figure 1 if one plots the average B number as a function of time, the above picture is revealed: a "J-shape". It turns out that this shape is a ubiquitous feature for the GDP evolution of many countries that went through a major "shock" in their economies like the collapse of the Soviet bloc for example.

the Internet works.

In global economics it was shown in [23] that the global economic system can be mapped into a modification of the "AB Model". In [18] the power law's distribution of the income and of the returns in the stock market were compared to give extraordinary fit with reality. One more paper was just written [7] on how could decision makers gain knowledge on the economic system they are in charge of under the light of this model.

In this section we show in some more details one specific example of how one can take the predictions resulting from these types of models and apply them to real life systems: The system we will discuss is the Polish economy following the collapse of the Soviet bloc. We chose to present the system with the aid of equation (6). In the present application, the index i of the equations in the system (6) ranges from 1 to 2945 and labels the economy of each of the 2945 counties composing Poland. Each equation represents the evolution of the economic activity W_i of the county i . More precisely, W_i is the number of enterprises per capita in the county i . The a_i 's represent endogenous growth rate of the county i and vary from county to county depending on local factors such as social capital, availability of natural resources or infrastructure. In fact the data indicate that the most important factor affecting the economic growth is the education level in the county. This dependence of a purely economical quantity on a social quantity is of great methodological importance and emphasizes in a dramatic way the importance of interdisciplinary studies (in this case economics, social science and physics). A recent work [43] led to a list of nine specific predictions resulting from the model. The data confirmed in a clear way the model predictions: Following the liberalization, the counties behaved in divergent ways: while most of the counties' economies plunged by factors of two, a few counties tripled their economic activity. This in turn lead to a quick increase in inequality between the counties. During the preceding (socialist) regime, all counties were allocated roughly the same amount of economic activity by the central government. Thus the counties with high post-liberalization growth rate represented initially a negligible part of the country GDP and could not avert the fast global decay. However, within a couple of years, following their dramatic growth, the fast developing counties became the economic force, driving up the GDP. Moreover, their influence expanded to the neighboring regions until, eventually the entire country reached an uniform growth rate. This is not to say that the economic activity per capita equalized. Quite contrary, in the asymptotic regime in which the growth of the weak regions was due to the diffusion of

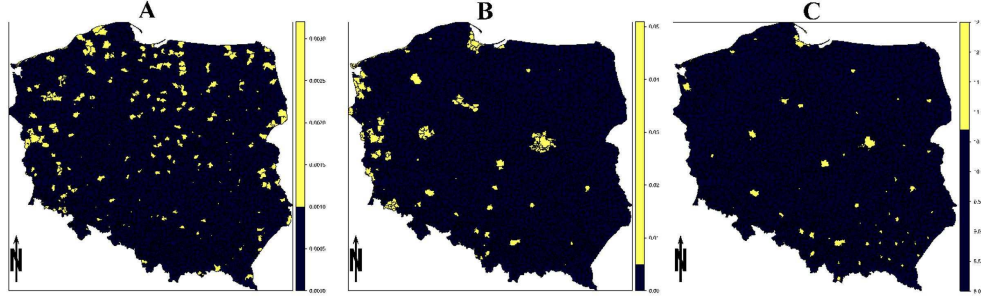


Figure 3: **The influence of Education on Economic Activity before and after liberalization** **3A** The number of enterprises per capita in each county in 1989. **3B** The number of enterprises per capita in each county in 1998. **3C** The years of education per capita in various counties in 1988. **3A** maps the number of the enterprises per capita in the year preceding the economic transition. This initial distribution does not display any spatial pattern: is very close to a uniform random (Poisson) distribution (similar to 1 at $t = 0$). **3B** After the liberalization there is a clear spatial pattern: the economic activity is concentrated around the singular growth centers which are strongly correlated with the education levels *before* the transition (**3C**). In the language of the "AB Model", the *A*'s - represent the education level, while the *B*'s represent the economic activity (enterprises per capita).

economic activity from the fast developing regions, the very wide differences in GDP per capita persisted and in fact increased. One can see in Figure 3 the spatial structure of the system, in $t = 0$ (year 1989, before the liberalization) and in 1994 and compare it with the social conditions (education level) that catalyzed the economic growth. The localization effect is very clear. In Figure 4 one sees how the generic prediction of the "J-shape" resulting from the "AB Model" is present in all of the formerly communist countries!

6 Future Directions

In his science fiction novel "Foundation" (1951), Isaac Asimov was playing with the idea of having a reliable predictions of the human society under the new scientific discipline he invented and called Psychohistory. In this novel he was dealing mainly with the fact that unlike other scientific disciplines - here,

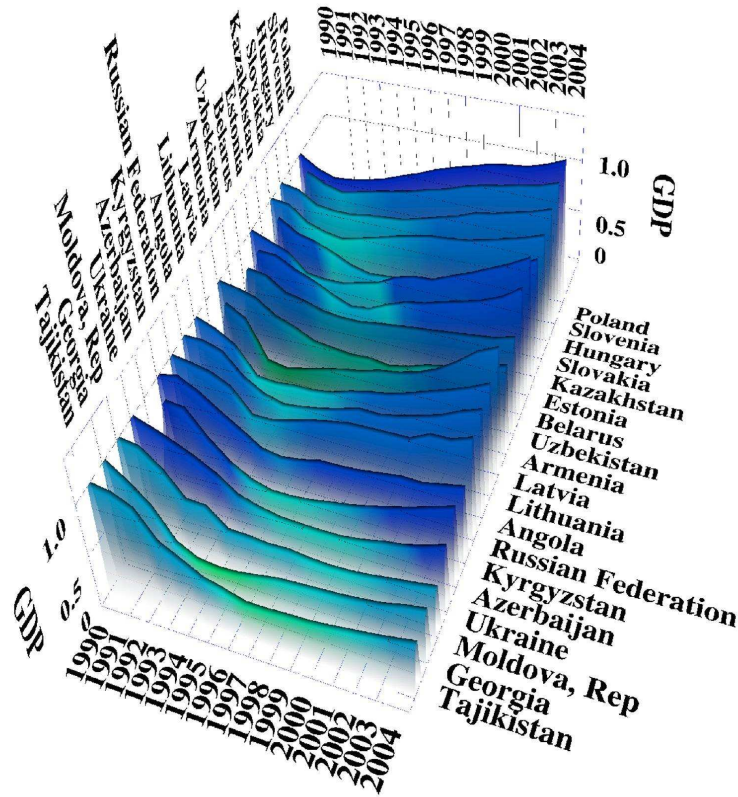


Figure 4: **The "J-curve" economic recovery after the liberalization of the Soviet Block** The GDP's of the Eastern European countries experienced strong decay immediately after the economic liberalization. This was generally followed by a growth period. The resulting pattern resembles the letter J which explains the name "J-curve". While the magnitude of the initial decay and the time and rate of recovery varies among the various countries, the J-shape is universal. The marked departure (exponential growth) of the Polish economic activity from an exponential decay extrapolated curve indicates that the classical global logistic framework cannot explain the observed pattern. The "AB Model", however does! (as can be seen in figure 2)

due to the fact that human beings are involved, they are able to read the predictions and by that changing them...Without entering this discussion, we feel that Asimov succeeded to put his finger on a very crucial point: unlike physics for example, there is no such tool that people can rely upon when trying to predict the possible future outcome of today's deeds. Many scientists that come from the natural sciences are very suspicious towards their colleagues from the "soft" sciences because of this reason. On the other hand the "soft" scientists are claiming that the problems that they deal with are far too complex to put into solvable equations. We do understand the positions of both sides, but feel that the time has come to try and close the cultural gap between the two. The methods of the accurate sciences have been improved dramatically since the availability of computer power, on the other hand the social sciences are able today (also due to computers and Internet) to measure many social indexes on a very wide scale and for many years. What is needed now is first to make efforts to quantify the observations that social scientists agree upon, and by that to create a set of so called "stylized facts". After having such list of qualitative and quantitative motifs that science agree they are present in reality - the road to having models that could be validated or invalidated by comparing their theoretical predictions to reality is closer than ever. We do believe that what was described in this paper are first steps towards Asimov's fantasy. Maybe this line of research will help us understand a little better the complex nature of human society.

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